Available online at www.sciencedirect.com

SCIENCE DIRECT®

Journal of APPLIED MATHEMATICS AND MECHANICS

ELSEVIER

www.elsevier.com/locate/jappmathmech

Journal of Applied Mathematics and Mechanics 69 (2005) 213–221

THE ROLE OF CROSS TERMS IN THE FLOW LAW FOR A GAS CONDENSATE MIXTURE IN A POROUS MEDIUM NEAR A WELL†

O. Yu. DINARIYEV

Moscow

e-mail: dinariev@mail.ru

(*Received* 17 *June* 2002)

The problem of the steady flow of a gas condensate mixture in a porous medium near a well is considered. Capillary forces and cross terms are taken into account in the flow law. It is shown that the problem splits into the purely physicochemical problem of capillary condensation and the problem of determining of the pressure field in the phases. The latter problem can have infinitely many solutions; the solution which is of the greatest interest in practice is separated out. An approximate solution in analytical form is found in the case of a small ratio of the gas and condensate viscosities. © 2005 Elsevier Ltd. All rights reserved.

1. INTRODUCTION

When modelling flows in porous media in the case of mixtures of the gas–liquid or liquid–liquid type when gravitational forces are unimportant, the generalized Darcy law for the average mass velocities of the phases [1]

$$u_n^a = -k\mu_n^{-1} f_n \partial_a p_n, \quad n = 1, 2 \tag{1.1}$$

is usually employed, where u_n^a are the flow rates of the phases, k is the absolute permeability, μ_n are the shear viscosities of the phases, p_n is the pressure in the phases and f_n are functions of the phase permeabilities, which depend on the saturation of one of the phases.

At the same time, modelling of the flow of a two-phase mixture in a capillary yields a result which differs from (1.1). For instance, when describing the flow of a multicomponent gas condensate mixture in an axially symmetric capillary using the density functional method [2] in the case when the condensate is a wetting phase, the following expressions for the average local velocities of the gas and the condensate are obtained

$$u_g = -R^2(\mu_g^{-1}\Sigma_1 + \mu_c^{-1}\Sigma_2)\frac{dp_g}{dx} - R^2\mu_c^{-1}\Sigma_3\frac{dp_c}{dx} \tag{1.2}$$

$$u_c = -R^2\mu_c^{-1}\Sigma_3\frac{dp_g}{dx} - R^2\mu_c^{-1}\Sigma_4\frac{dp_c}{dx} \tag{1.3}$$

where x is the coordinate along the axis of the capillary, R is the local value of the radius of the capillary, p_g and p_c are the pressures in the gas and in the condensate, μ_g and μ_c are the shear viscosities of the gas and the condensate, and Σ_1, Σ_2, Σ_3 and Σ_4 are dimensionless coefficients which depend on the local saturation of the condensate.

Relations (1.2) and (1.3) differ from expressions (1.1) in that there are cross terms: the flow of one phase depends on the pressure gradient in the other phase. This had been noted a long time ago and served as the basis for assumptions regarding the extension of flow law (1.1). Thus, for example, expression for the flow rates of the phases were proposed in the form [3]

---

†*Prikl. Mat. Mekh.* Vol. 69, No. 2, pp. 235–244, 2005.

doi: 10.1016/j.jappmathmech.2005.03.007

$$u_n^a = -k\mu_n^{-1}(f_{n1}\partial_a p_1 + f_{n2}\partial_a p_2), \quad n = 1, 2$$

where f_{nm} are functions of the phase permeabilities. The inadequacy of these last expression lies in the fact that the dependence on the viscosities of the phases does not correspond to the exact relations (1.2) and (1.3).

Steady flows of a gas condensate mixture near a well when the condensate is a wetting phase, which corresponds to the formulation of the problem described earlier in [2], will be considered below. The expressions with cross terms

$$u_g^a = -k((\mu_g^{-1}f_1 + \mu_c^{-1}f_2)\partial_a p_g + \mu_c^{-1}f_3\partial_a p_c) \tag{1.4}$$

$$u_c^a = -k\mu_c^{-1}(f_3\partial_a p_g + f_4\partial_a p_c) \tag{1.5}$$

are adopted for the flow rates of the phases, where $f_A = f_A(s_c)$ ($A = 1, 2, 3, 4$) are the relative phase permeability coefficients, which depend on the saturation of the condensate s_c.

The dependence on the viscosity in expressions (1.4) and (1.5) corresponds to expressions (1.2) and (1.3). Moreover, the coefficients of the cross terms in (1.4) and (1.5) are taken to be equal, which also corresponds to expressions (1.2) and (1.3) and to the well-known Onsager symmetry relations in non-equilibrium thermodynamics [3].

In order to satisfy all the requirements of non-equilibrium thermodynamics [4], the matrix of the coefficients in the flow law (1.4), (1.5)

$$\Lambda = \begin{Vmatrix} \mu_g^{-1}f_1 + \mu_c^{-1}f_2 & \mu_c^{-1}f_3 \\ \mu_c^{-1}f_3 & \mu_c^{-1}f_4 \end{Vmatrix} \tag{1.6}$$

must have non-negative eigenvalues. This property of the matrix (1.6) must not be associated with the actual values of the viscosities of the phases. We therefore adopt that following conditions which are imposed on the relative permeability coefficients and are sufficient for the non-negativity of the matrix (1.6)

$$f_A(s_c) \geq 0 \tag{1.7}$$

$$\Delta(s_c) = f_2(s_c)f_4(s_c) - f_3(s_c)^2 \geq 0 \tag{1.8}$$

Furthermore, we shall assume that the inequality

$$f_1(s_c) + f_4(s_c) > 0 \tag{1.9}$$

is satisfied, which means that at least one of the phases is mobile for any saturation value.

We shall also adopt the usual conditions for the relative phase permeability coefficients for limiting values of the saturation

$$f_1(0) = f_4(1) = 1 \tag{1.10}$$

$$f_1(1) = f_4(0) = f_2(0) = f_2(1) = f_3(0) = f_3(1) = 0 \tag{1.11}$$

It is well-known that relations (1.7)–(1.11) are also satisfied in problems with flow law (1.1). It should be noted that flow law (1.4), (1.5), which is used in this paper, is not the most general one. For instance, defining relations for the flows of multiphase, multicomponent fluids in the porous media, taking account of diffusion and heat conduction within the framework of interpenetrating continua, have been obtained by means of non-equilibrium thermodynamics.†

---

†KOLESNICHENKO, A. V. and MAKSIMOV, V. M., Thermodynamics of a multiphase, chemically active mixture. Darcy laws and diffusion, Preprint No. 52, The M. V. Keldysh Inst. Appl. Mat., Moscow, 1997; KOLESNICHENKO, A. V. and MASSIMOV, V. M. The generalized Darcy law as a consequence of the Stefan–Maxwell relations for a heterogeneous medium. Preprint No. 45, The M. V. Keldysh Inst. Appl. Math., Moscow, 1999.

A qualitative analysis of the solutions of the problem of the steady flow of a gas condensate mixture near a well in the case of flow law (1.4), (1.5) is given in Section 2. The case when the problem has a solution in analytical form is investigated in Section 3. The methods used in this paper correspond to the approach described earlier in [5–7] for investigating flows of a gas condensate mixture in the case of law (1.1).

2. THE GENERAL PROPERTIES OF THE SOLUTIONS

We will now consider an axisymmetric, isothermal, steady flow of a gas condensate mixture in a porous medium near an operational well. The system of equations for the conservation of the components

$$\partial_r(r(n_{ig}u_g^r + n_{ic}u_c^r)) = 0 \tag{2.1}$$

is satisfied, where r is the distance from the axis of the well, u_g^r and u_c^r are the flow rates of the gas and the condensates, and n_{ig} and n_{ic} are the molar densities of the components of the gas and the condensate respectively. Henceforth, the subscript i takes the values $1, \ldots, M$, corresponding to the number of the component.

Equations (2.1) have the M integrals

$$2\pi r h(n_{ig}u_g^r + n_{ic}u_c^r)) = -Q_i \tag{2.2}$$

where h is the thickness of the stratum and Q_i is the overall flow of the component of the gas condensate mixture with number i.

We shall assume that all the properties and characteristics of the gas and the condensate are defined by expressions which are common to the two phases [8]. Thus, $p_g = p(n_{ig})$ and $p_c = p(n_{ic})$ are the pressures in the gas and in the condensate, $\mu_s = \mu(n_{ig})$ and $\mu_c = \mu(n_{ic})$ are the shear viscosities of the gas and the condensate and $\kappa_{ig} = \kappa_i(n_{jg})$ and $\kappa_{ic} = \kappa_i(n_{jc})$ are the chemical potentials of the components of the gas and the components of the condensate.

The densities of the components of the gas and the components of the condensate are related by the conditions of local thermodynamic equilibrium

$$\kappa_{ig} = \kappa_{ic} \tag{2.3}$$

$$p_g - p_c = p_{\text{cap}} \tag{2.4}$$

Here, $p_{\text{cap}} = p_{\text{cap}}(s_c) \geq 0$ is the capillary pressure jump. It is assumed that $p_{\text{cap}} = p_{\text{cap}}(s_c) \geq 0$ is a smooth, monotonic, decreasing function of the saturation of the condensate s_c, $p_{\text{cap}}(1) = 0$.

We will now introduce the following notation: $Q = \sum_i Q_i$ is the overall flow of all of the components of the mixture, $n_g = \sum_i n_{ig}$, $n_c = \sum_i n_{ic}$ are the overall molar densities of the gas and the condensate respectively, and $c_{ig} = n_{ig}/n_g$ and $c_{ic} = n_{ic}/n_c$ are the concentrations of the components of the gas and the condensate respectively.

The relation

$$(1 - W)c_{ig} + Wc_{ic} = c_{i0} \tag{2.5}$$

$$W = -2\pi r h n_c u_c^r/Q = 1 + 2\pi r h n_g u_g^r/Q \tag{2.6}$$

which relates the concentrations in the phases, follows from Eqs (2.2).

The system of equations (2.3), (2.5) can be interpreted as a description of the decomposition of a mixture with composition c_{i0} into gas and condensate with compositions c_{ig} and c_{ic} and pressures in the phases p_g and p_c respectively. Here, W is the mole fraction of the condensate in the mixture. The problem of the decomposition of a mixture c_{i0} into a gas and a condensate is independent of the problem of the fluid flow in a porous medium and can be solved by the methods of chemical thermodynamics of the basis of a chosen equation of state for the mixture, for example [8]. For a fixed choice of c_{i0}, the solution, generally speaking, has the following functional form

$$\begin{aligned} &c_{ig} = c_{ig}(p_g, p_c), \quad n_g = n_g(p_g, p_c), \quad c_{ic} = c_{ic}(p_g, p_c) \\ &n_c = n_c(p_g, p_c), \quad W = W(p_g, p_c) \end{aligned} \tag{2.7}$$

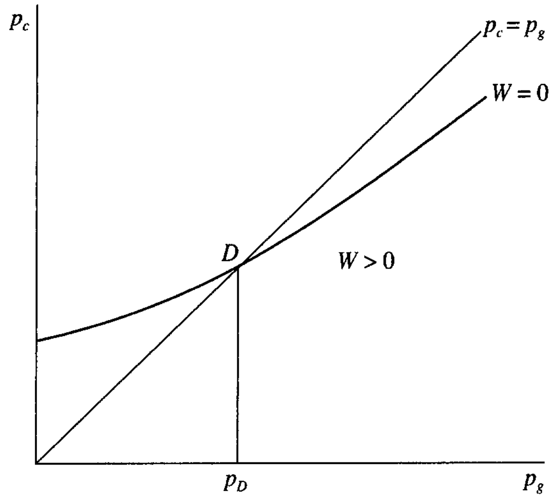

Fig. 1

The expressions for the partial derivatives $\partial W/\partial p_g$ and $\partial W/\partial p_c$ are presented in the appendix (Section 4). It is important that, when the function $W = W(p_g, p_c)$ vanishes for a certain pair of values p_g, p_c and there is no concentrate, $(\partial W/\partial p_c) < 0$ (see inequality (4.8)). Consequently, if the magnitude of p_g is reduced for a given value of p_c, condensate appears in the system.

We recall that, in the case of gas-concentrate deposits, the situation is typical when there is either no concentrate in the stratum a long way from the well or the quantity of condensate is insignificant and its mobility can be neglected. Hence the concentrations c_{i0} describe the gaseous phase of the mixture in the stratum. Close to the well, when the pressure p_g is reduced, the gaseous mixture becomes thermodynamically unstable and mobile condensate appears (the phenomenon of retrograde condensation). Taking account of the earlier remark concerning the nature of the dependence of the function W on p_c, it is possible to represent the position of the curve $W = 0$ qualitatively in the p_g, p_c plane (Fig. 1). Point D in Fig. 1 corresponds to the value of the pressure p_D at which the gaseous mixture becomes saturated (the dew point).

Instead of the spatial coordinate r, it is convenient to introduce the variable

$$\xi = Q(2\pi kh)^{-1}\ln(r/a) \tag{2.8}$$

The system of ordinary differential equations for the pressures in the phases then follows from Eqs (1.4), (1.5) and (2.6)

$$\frac{dp_g}{d\xi} = \frac{(1-W)f_4 n_g^{-1} - W f_3 n_c^{-1}}{\mu_g^{-1} f_1 f_4 + \mu_c^{-1}\Delta} \tag{2.9}$$

$$\frac{dp_c}{d\xi} = \frac{-(1-W)f_3 n_g^{-1} + W(\mu_c \mu_g^{-1} f_1 + f_2)n_c^{-1}}{\mu_g^{-1} f_1 f_4 + \mu_c^{-1}\Delta} \tag{2.10}$$

Equation (2.4), by virtue of the monotonicity of its right-hand side can be considered as an implicit definition of the saturation of the condensate in the form of a function of the difference of the pressures in the phases $s_c = s_c(p_g - p_c)$. After substituting this function into the system of equations (2.9), (2.10), a closed problem for determining the pressure $p_g = p_g(\xi), p_c = p_c(\xi)$ is obtained.

By virtue of the autonomy of the system of equations (2.9), (2.10), the single solution of the problem $p_g = p_g(\xi), p_c = p_c(\xi)$ generates the set of other solutions $p_g = p_g(\xi + \xi_0), p_c = p_c(\xi + \xi_0)$, where the magnitude of ξ_0 is arbitrary. Recalling the definition of the variable ξ (2.8), it can be concluded that the single solution which has been found enables one automatically to obtain the solution of the initial problem (2.1), (2.3), (2.4) for any other values of the parameters Q, k, h and a.

The solution of the problem of the flow of a gas condensate mixture in a porous medium must satisfy the usual boundary conditions, that is, the pressure on the end face of the well p_w and the pressure in the stratum p_0 are specified

$$p_g|_{r=r_w} = p_w, \quad p_g|_{r=r_0} = p_0 \tag{2.11}$$

where r_w is the radius of the well and r_0 is the radius of the feeding contour. The well production rate Q is determined from the solution of the problem, taking account of conditions (2.11).

It should be noted that the solution of the problem of the flow of a gas condensate mixture in a porous medium is not, generally speaking, uniquely defined by the boundary conditions (2.11). In fact, in the general case, the trajectories of system (2.9), (2.10) continuously fill the p_g, p_c plane and, consequently, they form a one-parameter family. By choosing any trajectory, the values of the parameter ξ, corresponding to the radii r_w and r_0, can be determined from conditions (2.11), and the parameters Q and a can then be calculated from Eq. (2.8). It is therefore possible to obtain an infinite number of solutions of the seepage problem of the fluid flow in a porous medium and it is necessary to have an additional criterion for selecting a unique solution.

We will not consider the problem of the non-uniqueness of the solution in greater detail. According to relation (2.6), the condensate is mobile only if $W > 0$. Consequently, if there is a non-zero threshold for the mobility of the condensate $s_{c1} > 0$, then, when $W > 0$, the saturation of the condensate must be higher than this threshold $s_c > s_{c1}$. When $W = 0$, the condensate is immobile (if it exists). Consequently, $f_2 = f_3 = f_4 = 0$ and Eq. (2.9) reduces to the form

$$\frac{dp_g}{d\xi} = \frac{\mu_g}{f_1 n_g} \tag{2.12}$$

In this case, there is an indeterminacy of the 0/0 type on the right-hand side of Eqs (2.9) and (2.10). In the case of Eq. (2.9), this indeterminacy is removable and division of the numerator and denominator on the right-hand side by f_4 enables one to prove the continuity at the point where a transition occurs from single-phase flow to two-phase flow. In Eq. (2.10), the 0/0 indeterminacy is not removable. In fact, the non-uniqueness of the solution is associated with this: system (2.9), (2.10) does not satisfy the Picard–Lindelöf existence and uniqueness theorem for ordinary differential equations [9].

From a physical point of view, two mechanisms for the occurrence of the non-uniqueness can be recognized.

1. Suppose $s_c = s_c(p_g)$ is the saturation value as a function of the pressure in the gas phase, which is obtained by eliminating the parameter p_c from the equations $W(p_g, p_c) = 0$ and (2.4). In other words, this is the limit value of the saturation of the condensate such that a mixture with a composition c_{i0} cannot decompose in the porous medium into a gas phase with a pressure p_g and a liquid phase with saturation in the interval $[s_c(p_g), 1]$. Then, the inequality $s_c(p_g) < s_{c1}$ can be satisfied for certain values of the pressure in the gas phase, which are higher than the saturation pressure $p_g > p_D$. In this case, a mobile gas phase with a composition c_{i0} can coexist in thermodynamic equilibrium with an immobile liquid phase, during which the saturation of the liquid phase lies in the interval $[s_c(p_g), s_{c1}]$. In spite of the fact that the liquid phase is immobile, it has an effect on the pressure field, since the coefficient $f_1 = f_1(s_c)$ exists on the right-hand side of Eq. (2.12).

The absence or presence of an immobile liquid phase does not contradict the basic equations of problem (2.1), (2.3), (2.4), and, hence, solutions with an arbitrary set of intervals of the radial coordinate r, in which an immobile liquid phase is present, are permissible in principle. However, if a flow near a well, which has been formed as a result of a gradual reduction in the pressure on the end face of the well, is considered, and there was initially no condensate in the stratum, then solutions with an immobile liquid must be excluded. On the other hand, if an immobile condensate exists from the outset in the stratum, it must also remain during the steady flow.

2. A jump in the saturation occurs at the point $\xi = \xi_t$ where a transition occurs from the domain $W = 0$ to the domain $W > 0$. As has been discussed above, when $\xi > \xi_t$, the saturation of the condensate is equal to zero or lies in the range $[s_c(p_g), s_{c1}]$. When $\xi < \xi_t$, the saturation of the condensate is strictly higher than the value s_{c1} and, simultaneously, strictly lower than the value s_{c2}, corresponding to the threshold of mobility for the gas. A value $s_{ct} = s_c|_{\xi = \xi_t - 0}$ can be arbitrarily chosen in the range $s_{c1} < s_{ct} < s_{c2}$ and this choice uniquely defines the solution in the region $\xi < \xi_t$.

As in case 1, if the steady flow is the result of a monotonic evolution of a stratum system in which there was initially no condensate, it is necessary to choose the solution with the minimum value of the saturation of the condensate. The corresponds to the boundary condition

$$s_{ct} = s_{c1} \tag{2.13}$$

So, the choice of the solutions must be based on the prehistory of the stratum system as a whole. We shall consider the solution with the smallest possible amount of condensate below. In the domain $W = 0$, there is no condensate and the pressure field is determined by Eq. (2.12) with $f_1 = 1$. In the domain $W > 0$, the solution of the problem is determined by system (2.9), (2.10) with boundary condition (2.13).

We recall that a small value of the ratio of the viscosities $(\mu_g/\mu_c) \sim 10^{-3} - 10^{-2}$ is a characteristic feature of gas condensate flows, and the asymptotic behaviour of the solutions in the limit of small (μ_g/μ_c) ratios is therefore of interest. We put $\mu_g = \varepsilon\mu_c\phi$, where ε is a small dimensionless quantity and ϕ is a finite dimensionless function. The system of equations (2.9), (2.10) becomes

$$\frac{dp_g}{d\xi} = \varepsilon\mu_c\phi\frac{(1-W)f_4 n_g^{-1} - Wf_3 n_c^{-1}}{f_1 f_4 + \varepsilon\phi\Delta} \tag{2.14}$$

$$\frac{dp_c}{d\xi} = \mu_c\frac{-(1-W)\varepsilon\phi f_3 n_g^{-1} + W(f_1 + \varepsilon\phi f_2)n_c^{-1}}{f_1 f_4 + \varepsilon\phi\Delta} \tag{2.15}$$

If the quantity f_1 in these equations remains finite and positive when $\varepsilon \to 0$, then the equations

$$\frac{dp_g}{d\xi} = 0, \quad \frac{dp_c}{d\xi} = \mu_c\frac{W}{n_c f_4}$$

are obtained in the leading approximation.

Consequently, as ξ becomes smaller, the pressure in the condensate falls monotonically which, by virtue of condition (2.4), implies that there is a monotonic decrease in the saturation and mobility of the condensate. However, it has been pointed out above that the condensate is necessarily mobile in the domain $W > 0$. This contradiction means that, in fact, the magnitude of f_1 tends to zero when $\varepsilon \to 0$, that is, the saturation s_c is identical with the magnitude of s_{c2}, which corresponds to the mobility threshold for the gas. At the same time, in accordance with (2.4), the magnitude of the capillary jump converges to a certain constant δ_{cap}, which is independent of the variable ξ.

The principal asymptotic form for f_1

$$\begin{gathered} f_1 = \varepsilon\phi((W^{-1}-1)f_{4*}n_c n_g^{-1} + f_{3*}(-1 + (W^{-1}-1)n_c n_g^{-1}) - f_{2*}) \\ f_{2*} = f_2(s_{c2}), \quad f_{3*} = f_3(s_{c2}), \quad f_{4*} = f_4(s_{c2}) \end{gathered} \tag{2.16}$$

can be calculated from Eqs (2.14) and (2.15) and the condition

$$d(p_g - p_c)/d\xi = O(\varepsilon)$$

Substituting expression (2.16) into the right-hand side of Eq. (2.14), we obtain an approximate equation for the pressure in the gas phase.

$$\frac{dp_g}{d\xi} = F; \quad F = \frac{\mu_c W}{n_c(f_{3*} + f_{4*})} + O(\varepsilon) \tag{2.17}$$

An expression for the function F with a higher order of accuracy can be found using the continuity of the right-hand side of Eq. (2.9) at the point where the transition from single-phase flow to two-phase flow occurs. Actually, since $W = 0$ when $\xi = \xi_t$, we obtain the expression

$$F = \frac{\mu_c W}{n_c(f_{3*} + f_{4*})} + \frac{\mu_g}{n_g} + O(\varepsilon^2) \tag{2.18}$$

Generally speaking, the quantities on the right-hand side of Eq. (2.17) are functions of the pressures in the gaseous and liquid phases. However, by virtue of the relation $p_c = p_g - \Delta p$, F can be considered as being a function of a single argument p_g, and it is therefore possible to find the solution when $\xi < \xi_t$ in the implicit form

$$\xi_t - \xi = \int_{p_g}^{p_t} \frac{dp_g}{F(p_g)} \tag{2.19}$$

where p_t is the pressure at the point ξ_t. The solution (2.19) must be joined with the solution when $\xi = \xi_t$, which is determined from Eq. (2.12).

The sum of the direct and cross coefficients of relative permeability appears in the final equation for the pressure in the gas (2.17). It can be seen that the effect of the cross coefficient leads to a reduction in the pressure gradient (that is, to a reduction in the pressure drawdown) for a specified production rate of the well.

3. THE SOLUTION IN ANALYTICAL FORM

We will now consider the flow of a gas condensate mixture, described by Eq. (2.12) in the single-phase domain and by Eq. (2.17) in the two-phase domain. In Eq. (2.12), the coefficient f_1 is assumed to be equal to unity. A solution in an analytical form can be obtained only if it is possible to represent the right-hand sides of Eqs (2.12) and (2.17) in the form of fairly simple expressions.

In Eq. (2.12), we will assume that the viscosity of the gas μ_g is constant and we will take the equation of state in the form

$$p_g = ZRTn_g$$

where Z is a constant, dimensionless coefficient which is called the coefficient of hypercompressibility, R is the universal gas constant and T is the temperature in the stratum. It is then easy to integrate Eq. (2.12).

$$p_g^2 - p_t^2 = 2\mu_g ZRT(\xi - \xi_t) \tag{3.1}$$

In Eq. (2.17) we assume that the viscosity of the condensate μ_c and the density of the condensate n_c are constants. It has been noted in [5] that, when there are no capillary forces, the function of the mole fraction of the condensate W is well-described by the expression

$$W = a_1 p + a_2/(p - a_3) \tag{3.2}$$

Here, a_1, a_2, a_3 are positive constants which are determined with respect to two points, the dew point and the point where the function W is a maximum [6]. In other words, these constants must be found from the system of equations

$$W(p_D) = 0, \quad W(p_m) = W_m, \quad \frac{dW}{dp}(p_m) = 0 \tag{3.3}$$

where p_m is the pressure at which the function W reaches its maximum value W_m. The system of equations (3.3) has two sets of solutions when $p_m/p_D > 0.75$. The best approximation of the experimental contact condensation curves is attained for the set of coefficients with the smaller value of a_3.

It is necessary to modify expression (3.2) when account is taken of capillary forces. The simplest generalization of the expression for the mole fraction of the condensate can be obtained by adding a term which is linear in the pressure in the liquid phase

$$W = a_1 p_g + a_2/(p_g - a_3) + a_4(p_g - p_c) \tag{3.4}$$

to the right-hand side of (3.2). Here, a_4 is a positive coefficient which must be selected from the experimental data on capillary condensation. It is clear that expression (3.4) satisfies inequality (4.8) (see Section 4 below). In the case of a fixed capillary jump $p_g - p_c = \delta_{\text{cap}}$, the function W vanishes at two points: p_* and p_t and the inequalities $p_* < p_D < p_t$ are satisfied. Therefore, the formula

$$W = a_1(p_g - p_*)(p_g - p_t)/(p_g - a_3)$$

holds.

Apart from terms $O(\varepsilon^2)$, Eq. (2.17) therefore reduces to the form

$$\frac{dp_g}{d\xi} = \frac{a_1\mu_c(p_g - b_1)(p_g - b_2)}{n_c(f_{3*} + f_{4*})(p_g - a_3)}$$

$$b_1 = p_* + b_0(p_* - a_3), \quad b_2 = p_t - b_0(p_t - a_3)$$

$$b_0 = \frac{\mu_g ZRTn_c(f_{3*} + f_{4*})}{a_1\mu_c p_t(p_t - p_*)}$$

In accordance with formula (2.18), the implicit solution of the problem in two-phase domain

$$\frac{b_1 - a_3}{b_1 - b_2}\ln\frac{p_t - b_1}{p_g - b_1} + \frac{b_1 - a_3}{b_1 - b_2}\ln\frac{b_2 - p_t}{b_2 - p_g} = \frac{\mu_c(\xi_t - \xi)}{n_c(f_{3*} + f_{4*})} \tag{3.5}$$

is obtained from this.

Relations (3.5) shows that the cross coefficient of the relative phase permeability f_3 is present in the solution in a sum with the coefficient f_4 and leads to an increase in the mobility of the condensate. The coefficient f_2 drops out from the final solution.

4. APPENDIX

A derivative of the differential relations which the functions (2.7) satisfy is presented below. Summation is carried out over repeated subscripts corresponding to the numbers of the components. It is convenient to use the thermodynamic potential (the Gibbs energy) of the mixture $\psi = \psi(p, c_i) = c_i\kappa_i$. Since the concentrations satisfy the normalization condition $c_1 + c_2 + \ldots + c_M = 1$, the function ψ actually depends on the concentrations $c_1, \ldots, c_{M-1}$. We shall use the Greek subscripts $\alpha, \beta = 1, \ldots, (M-1)$, which correspond to this shortened set of concentrations. We also introduce the following notation for the partial derivatives: $\psi_\alpha = \partial\psi/\partial c_a$, $\psi_p = \partial\psi/\partial p$.

We recall the well-known relations which the thermodynamic potential satisfies [5]

$$\psi_{,\alpha} = \kappa_\alpha - \kappa_M, \quad \psi_{,p} = n \tag{4.1}$$

Taking account of relations (4.1), we can rewrite the phase equilibrium condition (2.3) in the form

$$\psi_{,\alpha g} = \psi_{,\alpha c} \tag{4.2}$$

$$\psi_g - c_{\alpha g}\psi_{,\alpha g} = \psi_c - c_{\alpha c}\psi_{,\alpha c} \tag{4.3}$$

Differentiation of the $2M$-1 independent equalities (2.5), (4.2), and (4.3) leads to $2M$-1 linear equations relating the $2M + 1$ differentials of the concentrations $dc_{\alpha g}$, $dc_{\alpha c}$, of the pressures dp_g, dp_c and the mole fraction of the condensate dW. From these equations, the differentials $dc_{\alpha g}$, $dc_{\alpha c}$ and dW can be expressed as linear functions of the differentials dp_g, dp_c.

In order to write these expressions in a compact form, it is convenient to change to an abbreviated matrix description in the case of quantities which depend on the Greek subscripts. Thus, for the vectors $\eta = (\eta_\alpha)$, $\eta' = (\eta'_\alpha)$ we define a scalar product using the formula $(\eta, \eta') = \eta_\alpha\eta'_\alpha$ and, in addition, introduce the notation

$$c_g = (c_{\alpha g}), \quad c_c = (c_{\alpha c})$$

$$A_g = (A_{\alpha\beta g}) = (\psi_{,\alpha\beta g}), \quad B_g = (B_{\alpha g}) = (n_g^{-2}n_{,\alpha g})$$

$$A_c = (A_{\alpha\beta c}) = (\psi_{,\alpha\beta c}), \quad B_c = (B_{\alpha c}) = (n_c^{-2}n_{,\alpha c})$$

$$b_g = n_g^{-1} + (B_g, c_g), \quad b_c = n_c^{-1} + (B_c, c_c)$$

$$\Gamma_g = WA_g, \quad \Gamma_c = (1 - W)A_c$$

Note that the two phases, gas and condensate, are assumed to be locally thermodynamically stable, and the matrices A_g and A_c are therefore positive.

Differentiating relations (2.5), (4.2) and (4.3), we obtain the equations

$$\begin{aligned} Wdc_c + (1-W)dc_g + (c_c - c_g)dW &= 0 \\ A_g dc_g - B_g dp_g &= A_c dc_c - B_c dp_c \\ (c_g, A_g dc_g) - b_g dp_g &= (c_c, A_c dc_c) - b_c dp_c \end{aligned} \tag{4.4}$$

Eliminating the differentials of the concentrations from these equations, we obtain an expression for the differential of the mole fraction of the condensate

$$dW = \omega_0^{-1}(\omega_g dp_g - \omega_c dp_c) \tag{4.5}$$

$$\omega_0 = \frac{(c_g - c_c, (\Gamma_g - \Gamma_g(\Gamma_g + \Gamma_c)^{-1}\Gamma_g)(c_g - c_c))}{W(1-W)}$$

$$\omega_g = b_g - (\Gamma_g c_g + \Gamma_c c_c, (\Gamma_g + \Gamma_c)^{-1} B_g)$$

$$\omega_c = b_c - (\Gamma_g c_g + \Gamma_c c_c, (\Gamma_g + \Gamma_c)^{-1} B_c)$$

Expressions for the differentials of the concentrations

$$\begin{aligned} dc_g = {} & (\Gamma_g + \Gamma_c)^{-1}((WB_g + \omega_0^{-1}\omega_g A_c(c_c - c_g))dp_g - \\ & -(WB_c + \omega_0^{-1}\omega_c A_c(c_c - c_g))dp_c) \end{aligned} \tag{4.6}$$

$$\begin{aligned} dc_c = {} & (\Gamma_g + \Gamma_c)^{-1}(((W-1)B_g + \omega_0^{-1}\omega_g A_g(c_c - c_g))dp_g - \\ & -((W-1)B_c + \omega_0^{-1}\omega_c A_g(c_c - c_g))dp_c) \end{aligned} \tag{4.7}$$

are obtained in a similar manner.

Relations (4.5)–(4.7) can be used in numerical algorithms to determine the functions $c_{ig} = c_{ig}(p_g, p_c)$, $c_{ic} = c_{ic}(p_g, p_c)$, $W = W(p_g, p_c)$ by the method of small increments in the pressures in the phases.

The behaviour of the differential dW at points where $W = 0$ is of special interest. For this case, the expression

$$\frac{\partial W}{\partial p_c} = -n_c^{-1}((c_c - c_g), A_g(c_c - c_g))^{-1} < 0 \tag{4.8}$$

is obtained from formula (4.5).

By virtue of the continuity, inequality (4.8) still holds in a certain neighbourhood of the curve $W = 0$. Hence, in the case of a small mole fraction of condensate, a drop in the pressure in the condensate gives rise to an increase in the amount of concentrate (capillary condensation).

This research was supported by the international oil and gas company Schlumberger Oilfield Services.

REFERENCES

1. NIKOLAYEVSKII, V. N., BONDAREV, E. A., MIRKIN, M. I. *et al.*, *The Motion of Hydrocarbon Mixtures in a Porous Medium*, Nedra. Moscow, 1968.
2. DINARIYEV, O. Yu., A description of the flow of a gas condensate mixture in an axisymmetric capillary by the density functional method. *Zh. Prikl. Mekh. Tekh. Fiz.*, 2003, **44**, 1, 101–107.
3. KURBANOV, A. K., The equations of motion of two-phase fluids in a porous medium. In *The Theory and Practice of Oil Extraction*. Nedra, Moscow, 1968, 281–286.
4. GROOT, S. R. De and MAZUR, P., *Non-equilibrium Thermodynamics*. North-Holland, Amsterdam; 1962.
5. DINARIYEV, O. Yu., Multicomponent steady seepage flows with phase transitions. *Prikl. Mat. Mekh.*, 1994, **58**, 6, 78–85.
6. BABEIKO, A. Yu. and DINARIYEV, O. Yu., Modelling of retrograde condensation accompanying steady-state radial seepage. *Izv. Ross. Akad. Nauk. MzhG.*, 1994, 6, 92–97.
7. DINARIYEV, O. Yu., Retrograde condensation accompanying radial seepage. *Inzh.-Fiz. Zh.*, 1994, **67**, 1–2, 98–102.
8. GUREVICH, G. P. and BRUSILOVSKII, A. I., *Reference Book on the Calculation of the Phase State and Properties of Gas Condensate Mixtures*. Nedra, Moscow. 1984.
9. HARTMAN, PH. *Ordinary Differential Equations*. Wiley, New York, 1964.

Translated by E.L.S.